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ARTÍCULO

In/Stability under Ideal Income Tax and Ideal Consumption Tax

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KEYWORDS:
Progressive income tax; progressive consumption tax; multiplier-accelerator model; balanced-budget rule

Abstract: This article compares the stabilization properties of a comprehensive income tax and a broad-based consumption tax within a typical multiplier-accelerator model of the business cycle. It is found that: (i) a progressive consumption tax is more stabilizing than a progressive income tax in the absence of a budget rule, (ii) the income tax is as stabilizing in the presence of such a rule as consumption tax is in its absence, (iii) under reasonable tax rates, both taxes are equally stabilizing under a balance-budget rule, (iv) both at an output cost relative to the case in which there is no such rule. Also, (v) in the absence of the balanced-budget rule, the consumption tax is superior from the viewpoint of fixed-point output under equal for both taxes revenue, (vi) but no clear-cut results may be obtained in the presence of this rule, (vii) which rule complicates in addition the matter of tax equivalence considerably, making it a matter of nonlinear taxation even in the simple analytical framework of this paper.

CÓDIGOS JEL:
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PALABRAS CLAVE:
Impuesto sobre la rentaprogresivo; impuesto progresivo al consumo; modelo multiplicador-acelerador; regla de presupuesto equilibrado

Resumen: Este artículo compara las propiedades de estabilización de un impuesto a la renta integral y un impuesto al consumo de base amplia dentro de un modelo típico multiplicador-acelerador del ciclo económico. Se encuentra que: (i) un impuesto al consumo progresivo es más estabilizador que un impuesto a la renta progresivo en ausencia de una regla presupuestaria, (ii) el impuesto a la renta es tan estabilizador en presencia de dicha regla como lo es el impuesto al consumo en su En ausencia, (iii) bajo tasas impositivas razonables, ambos impuestos se estabilizan igualmente bajo una regla de equilibrio presupuestario, (iv) ambos a un costo de producción en relación con el caso en el que no existe tal regla. Además, (v) en ausencia de la regla de presupuesto equilibrado, el impuesto al consumo es superior desde el punto de vista de la producción de punto fijo bajo neutralidad de los ingresos tributarios, (vi) pero no se pueden obtener resultados claros en presencia de esta regla, (vii) la cual complica además la cuestión de la equivalencia tributaria considerablemente, convirtiéndola en una cuestión de tributación no lineal incluso en el marco analítico simple de este artículo.

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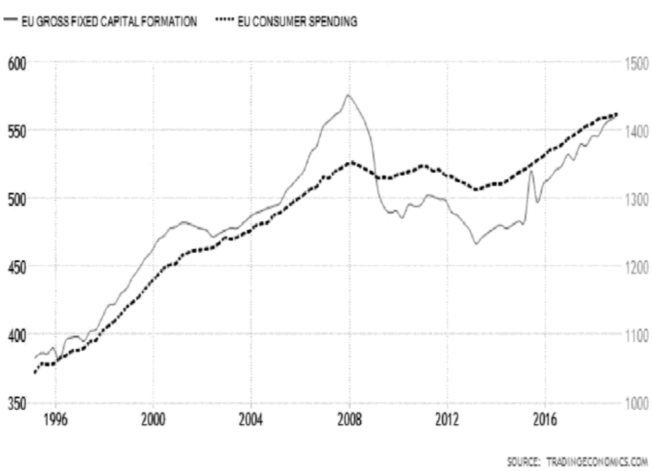
1. Introduction

One proposal to reform direct taxation has been the replacement of income taxation by a tax based on consumption expenditure. The debate on the applicability of consumption tax has been slow because opinions vary, and this disagreement reflects on the relative advantages of each tax in practice (Carroll et al. 2011). Nevertheless, there has been a lively debate about the stability implications of the two taxes defined comprehensively, i.e. as a comprehensive income tax on income from any source, and as a broad-based-consumption tax on comprehensive income minus all kinds of savings (Ghilardi and Rossi 2014). One reason for the concern for the nexus between tax reform and instability owes to the close relationship between investment and consumption. An example of this relationship is presented through Fig. 1, which plots the time paths of the two variables in Eurozone from 1995 to 2017, given the shocks from the introduction of Euro in early 2002, from Lehman Brothers crisis in 2008-2009, and from the subsequent debt problem in the south of the zone: Investment tracks consumption and although a consumption tax would not tax the return to new investment, it would "discourage the very thing that drives the economy: growth in consumption" (Boyer and Russell 1995, 367)...Or, not, by comparison to the effects on stability and growth from the way income tax is administered? (Gale and Samwick2016).

Fig. 1. Investment and Consumption in Eurozone: 1995-2019



More specifically, the literature on instability, which employs typically a real business cycles model, investigates whether the consumption tax can relax the destabilizing effect of balanced-budget-rule income taxation due to self-fulfilling expectations (Nishimura et al. 2013, Nourry et al. 2013)[1].The conclusion is that the consumption is subject to the same expectations problem unless an open economy model with free capital mobility is contemplated (Meng and Xue 2015).Given capital and labor, an expected increase in the income tax rate, will lower the expected return on capital and the marginal utility of income, reducing thereby current labor supply and production, and making the government raise the tax rate now due to balanced-budget rule regardless the base of the tax. In the case of consumption tax, the result is the same but because the expected increase in the tax rate will lower expected consumption, increasing thereby current consumption at the expense of current labor supply. Yet, this reduction in labor need not occur if there can be borrowing from abroad. When debt accumulation is allowed in the closed model with new Keynesian frictions, the result is found to depend as anticipated on the fiscal-monetary policy mix, on the equilibrium debt-output ratio, and on the intertemporal elasticity of substitution in consumption (McKnight 2017).

This paper compares the stabilization properties of a comprehensive income tax and a broad-based consumption tax within the context of the typical multiplier-accelerator model, abstracting from the matter of expectations, which Westerhoff (2006) finds them to be destabilizing in such a model, anyway. This approach differs from Giannitsarou's (2007) work which compares a mix of consumption and incomes taxes with a capital tax in support of the better stabilization prospects of this mix in a closed economy. It also abstracts from McKnight's (2017) focus on public debt and monetary policy, because in reality the issue of the debt is related intimately to open economy considerations. More importantly, there appears to be a contradiction between these two approaches since a time-consistent optimal policy with history-dependent strategies prescribes zero long-run capital taxation; and "a non-balanced budget constraint is key in obtaining this result, as it allows the government to increase its assets until the lack of commitment is no longer binding" (Ortigueira 2012, 4)[2]. Therefore, although the simplicity with which the topic under investigation is put herein is surprisingly novel by itself, this simplicity is dictated for methodological reasons, too. It is a methodology which would remain fruitless if the presence expectations were acknowledged as well, because as Westerhoff (2006) remarks, they should be based on an extrapolative-cum-regressive expectation formation rule, which herein would lead to comparison of chaotic behaviors.

It is thus found that when there is not any budget rule, it is the progressive consumption rather than income tax that is more stabilizing, while the income tax in the presence of such a rule is equally stabilizing as the consumption tax is in its absence. Under reasonable tax rates and a balanced-budget rule, no difference in stabilization between the two taxes is detected, though stabilization entails an output cost by comparison to stabilization in the absence of such rule. Also, given tax revenue equivalence and a balanced-budget rule, the consumption tax is superior from the viewpoint of fixed point output. Yet, no definite results may be obtained in the presence of this rule, which rule complicates the matter of tax equivalence significantly, too, making it a matter of nonlinear taxation even in the simple analytical environment of this article.

The comparison of the stabilization properties of a comprehensive income tax and a broad-based consumption tax is made formally in the next section. Section 3 concludes this paper with a discussion of the results.

2. Formal Considerations

Let Y, C, I, G and T denote national economyincome, consumption, investment, government expenditure, and tax revenue, correspondingly. A version of the standard model with a linear progressive income tax (Turnovsky 1977) has as follows:

$$Y = C + I + G \quad [1]$$
$$C = \theta(Y_{-1} - T_{-1}) \quad [2]$$

$$I = I_0 + \varepsilon(C - C_{-1}) \quad [3]$$
$$T = -\tau_0 + \tau Y \quad [4]$$

where the subscript " -1 " denotes one period lag. Coefficients θ and ε are positive while I_0 and τ_0 are the autonomous parts of [3] and [4], respectively[3]. $\theta < 1$ is the marginal propensity to consume out of income net of taxes, and $\varepsilon < 1$ is a proportion of the change in consumption (and hence, of net income). Inserting [2] in [3] and the result along with [2] and [4] in [1], yields the linear second-order difference equation:

$$Y = I_0 + G_0 + \theta\tau_0 + [\theta(1+\varepsilon)(1-\tau)]Y_{-1} - \theta\varepsilon(1-\tau)Y_{-2} \quad [5]$$

for some $G = G_0$, where subscript " -2 " designates second period lag. The fixed point of this equation is:

$$\hat{Y} = \frac{I_0 + G_0 + \theta\tau_0}{1-\theta(1-\tau)} \quad [6]$$

When the homogeneous equation has only one general solution, the Appendix shows that the stability condition, is:

$$\tau \geq 1 - \frac{2}{\theta(1+\varepsilon)} \quad [7]$$

which is satisfied for the values of τ in the area below the curves of Fig. 2 where $y \equiv \tau$ and $x \equiv \varepsilon$. These curves depict the case of the equality sign for three different θ 's: $\theta = 0.75, \theta = 0.85$ (dotted line in the middle), and $\theta = 0.95$ (upper line). The equality sign holds for stable motion along a uniform cycle.

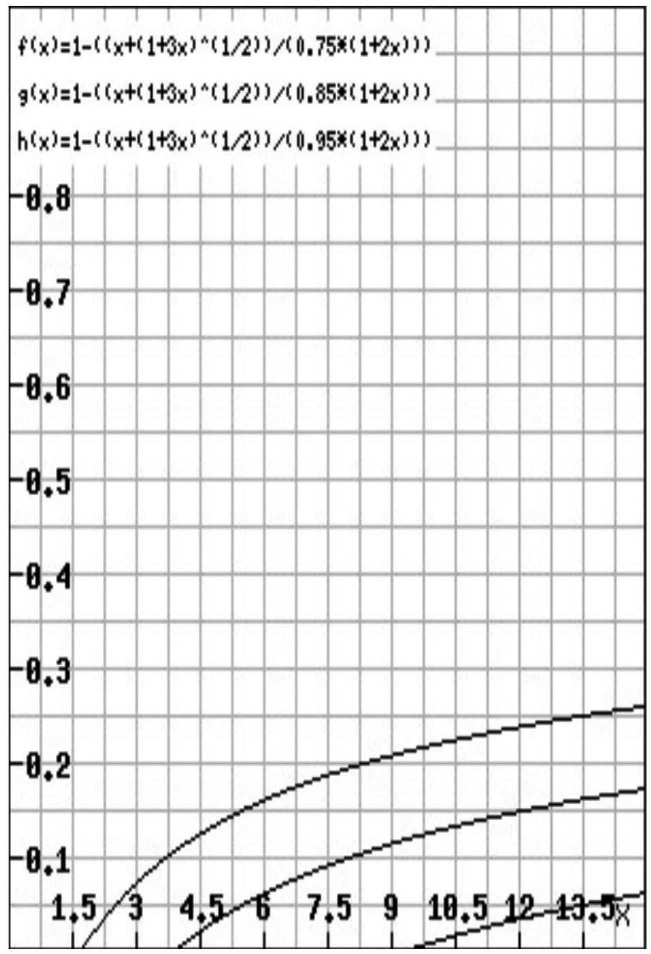
Fig. 2: Stability under an income tax with one real root

If there are two real solutions, the Appendix shows that we can accept only the one with associated stability condition:

$$\tau > 1 - \frac{\varepsilon + \sqrt{1+3\varepsilon}}{\theta(1+2\varepsilon)} \quad [8]$$

This condition pertains to τ 's above the lines depicted in Fig. 3. Again, $y \equiv \tau$ and $x \equiv \varepsilon$, in this Figure, and there are three different θ 's: $\theta = 0.75, \theta = 0.85$ (dotted line in the middle), and $\theta = 0.95$ (upper line). Stable motion along a uniform cycle would be the case if [8] held with the equality sign, but it does not.

Fig.3: Stability under an income tax with two real roots



These are results confirming the stabilizing role of linear progressive taxation.

Next, consider the linear progressive consumption tax:

$$T_c = -t_0 + tC \quad [9]$$

so that:

$$(1-t)C = \theta(t_0 + Y_{-1}) \quad [10]$$

Presumably, the propensity to consume need not be the same as under an income tax, but we keep it unchanged for comparison purposes. The second-order difference equation becomes now:

$$Y = \frac{\theta(1+\varepsilon)Y_{-1} - \varepsilon\theta Y_{-2} + (I_0 + G_0 + \theta t_0)(1-t)}{(1-t)} \quad [11]$$

with fixed point at:

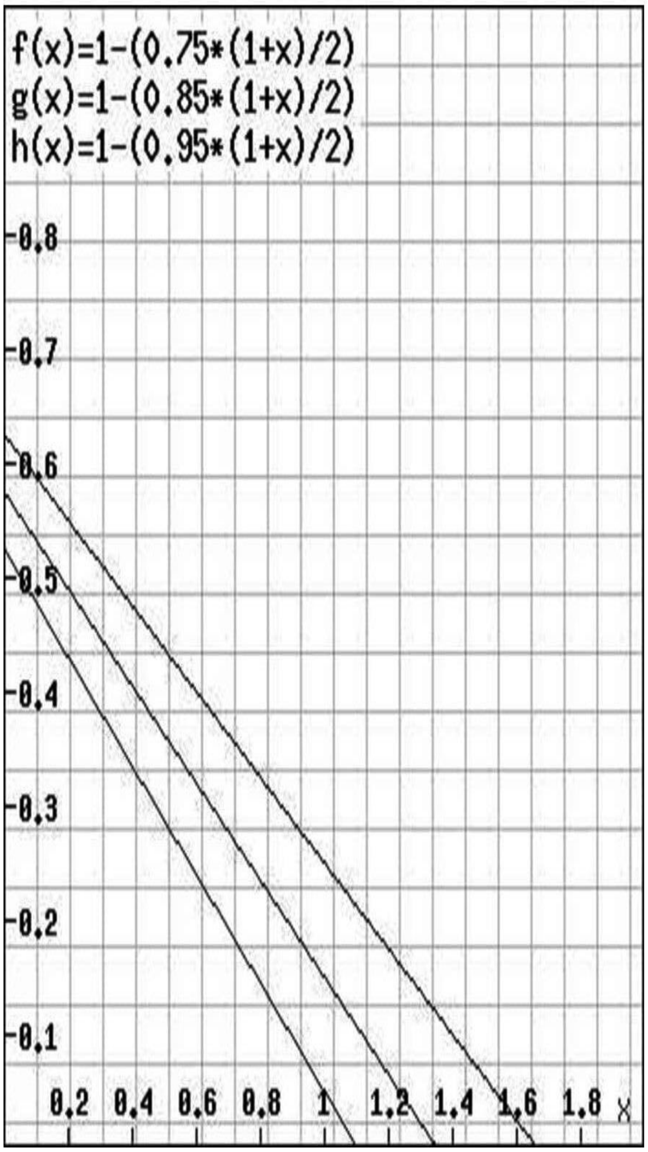
$$\tilde{Y} = \frac{(I_0 + G_0 + \theta t_0)(1-t)}{1-\theta-t} \quad [12]$$

When the homogeneous equation has only one general solution, the Appendix shows that the stability condition, is:

$t \geq 1 - \frac{\theta(1+\varepsilon)}{2}$ [13]

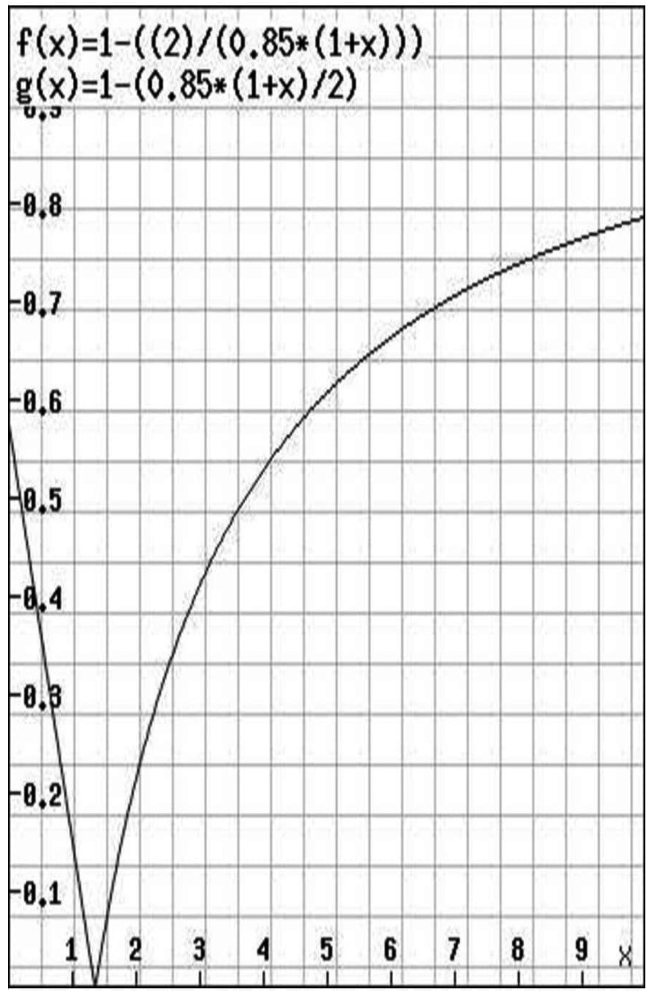
which is satisfied for the values of t in the area below the curves of Fig. 4 where $y \equiv t$ and $x \equiv \varepsilon$. These curves depict the case of the equality sign for three different θ 's: $\theta = 0.75, \theta = 0.85$ (dotted line in the middle), and $\theta = 0.95$ (upper line). The equality sign holds for stable motion along a uniform cycle.

Fig. 4: Stability under a consumption tax with one real root



The difference between Fig. 4 and Fig. 2 is striking, because the fraction in [13] is the opposite from that in [7]. The stability conditions implied by the two taxes are quite different. Consider Fig. 5where $\theta = 0.85$. The area above and to the right of the negatively sloped line is consistent with stability under a consumption tax. From the viewpoint of policymaking, one need not worry about the effects of taxation on stability once the accelerator takes on values higher than 1.3 or 1.4 and the tax is one on consumption. But, only the area above and to the left of the positively sloped curve is consistent with stability under an income tax. Of course, the area above the line and the curve and below $y = 1$ satisfies stability under either tax.

Fig. 5: Stability with one real root under either tax



In the case of two real solutions to the homogeneous equation,we can acceptonly the solution with associated stability condition:

$t < 1 - \theta$ [14]

If s is the saving rate so that $s + t + \theta = 1 \Rightarrow 1 - \theta = s + t$, [14] becomes $s > 0$; that is, all that stability requires is the presence of some savings. The condition $t > 1 - \theta$ associated with the other solution is not acceptable because it would imply that the denominator of [12] is negative. This denominator would be zero if [14] held with an equality sign and therefore, there cannot be stable motion along a uniform cycle as was the case with [8], too. Nevertheless, [14] is independent of the coefficient of accelerator, which coefficient according to [8] would be decisive for stability under income taxation. The general conclusion confirms the proposition that a consumption base tax is much superior for stability relative to an income tax.

Moreover, it is superior from the viewpoint of fixed-point output under equal for both taxes revenue. In general:

$\tilde{Y} \geq \hat{Y} \Rightarrow \tilde{Y} \geq \frac{\tau_0 - t_0}{t + \tau(1-t)}$ [15]

which would be true with certainty if $t_0 \geq \tau_0$ given that the denominator is positive. But, $\tilde{Y} > \hat{Y}$ for sure under equal tax revenue, $\tau_0 - t_0 = \tau\hat{Y} - t\tilde{Y}$, because when this equality is inserted in [15], yields:

$$\frac{\tilde{Y}-\hat{Y}}{\tilde{Y}} \geq t \quad [16]$$

and, since $t > 0$, it follows that $\tilde{Y} - Y \rightrightarrows > 0$, always.

Now, note that balanced-budget-rule taxation upsets this picture of the two taxes dramatically. Substituting $G = T = -\tau_0 + \tau Y$ in [5] and solving again for Y, yields:

$$Y = \frac{I_0-\tau_0(1-\theta)}{1-\tau} + \theta(1+\varepsilon)Y_{-1} - \theta\varepsilon Y_{-2} \quad [17]$$

with fixed point at:

$$\bar{Y} = \frac{I_0-\tau_0(1-\theta)}{(1-\theta)(1-\tau)} \quad [18]$$

with the only admissible solution to the homogeneous equation, being associated with the stability condition:

$$1-\theta > 0$$

i.e. that the saving rate is positive (see Appendix). A balanced-budget rule takes away instability from the multiplier-accelerator model completely, suffices that there is some positive saving. The balanced-budget-rule progressive income tax is as stabilizing as the progressive consumption tax under [14].Once the private-public sector interaction is subject to this rule, the accelerator ceases to play any stabilizing role. This improvement of stability comes at an output cost, because:

$$\bar{Y} < \hat{Y} \Rightarrow \tau_0(\theta-1) < I_0 + G_0(1-\theta)(1-\tau)$$

which is true since the $\tau_0(\theta-1) < 0$ and $I_0 + G_0(1-\theta)(1-\tau) > 0$. And, since $\hat{Y} < \tilde{Y}$, it follows that $\bar{Y} < \tilde{Y}$, too: $\bar{Y} < \hat{Y} < \tilde{Y}$.

In so far as the difference equation in connection with the consumption tax is concerned:

$$Y = I_0 - t_0(1-\theta) + \frac{\theta(1+\varepsilon)Y_{-1}-\theta\varepsilon Y_{-2}}{1-t} \quad [19]$$

the fixed point is:

$$\bar{Y} = \frac{[I_0-t_0(1-\theta)](1-t)}{1-\theta-t} \quad [20]$$

and the stability condition at $\varepsilon = 1$ is:

$$t > 1-\theta = s$$

which is not as restrictive as at first sight appears to be when compared to the condition in the absence of the balanced-budget rule(see Appendix) or to the condition under an income tax in the presence of such rule. Simply, a $\tau = s + \epsilon$, with a very small c, is a pragmatic rate, in which case both taxes, $t = \tau = s + \epsilon$, would be equally stabilizing. Yet, both taxes cannot have the same multiplier because if they did, we would have that $t = \tau(1-\theta) < (1-\theta)$ given $\tau < 1$. The multiplier in connection with the consumption tax should exceed that under the income tax if stability is required: $t > \tau(1-\theta)$.

The general conclusion is that from the viewpoint of stability, income taxation is preferable to consumption taxation under a balanced-budget rule, but the opposite is true in the absence of this rule. McKnight (2015,2017) would argue that this might be true on certain occasions but the literature has typically found that consumption taxes are better than income taxes under balanced-budget rules. The next section points out that the disagreement here with the literature owes to the identification of balanced-budget rules with the budget constraint and to the comparison of the two taxes in terms of output loss rather than output volatility. Moreover, comparing [20] with [12], it is clear that there is an output loss in the case of consumption taxation, too:

$$\bar{Y} < \tilde{Y} \Rightarrow -t_0 < G_0$$

which is true. Nevertheless, the comparison: $\bar{\bar{Y}} > \bar{Y}$, amounts to:

$$\tau_0(1-\theta)(1-\theta-t) + t\theta I_0 > (1-\theta)(1-t)[\tau I_0 + t_0(1-\theta)(1-\tau)]$$

which may or may not be true while as the Appendix shows, complicated nonlinear taxation is required in order to have: $-\tau_0 + \tau\bar{Y} = -t_0 + t\bar{\bar{Y}}$. The simplest case is under very strict assumptions that:

$$t > \tau > \frac{I_0(2I_0+5\tau_0\theta+\tau_0-3\theta I_0-2\tau_0\theta^2-2t_0)}{\tau_0^2+\theta\tau_0^2+\theta^2 I_0^2+4I_0^2+6\tau_0\theta I_0-4\theta I_0^2-4\tau_0 I_0-4\tau_0\theta^2 I_0}$$

Fanti and Manfredi (2003) demonstrate that nonlinear progressive income taxation can be destabilizing within the multiplier-accelerator model. If we accept this to be true under a balanced-budget rule too, or if nonlinear progressive consumption taxation is destabilizing as well, tax equivalence should not be pursued, which are matters that remain to be investigated.

3. Concluding Remarks

The balanced-budget constraint with an income tax has been found to be destabilizing not only within the context of real business cycle modeling but also within the multiplier-accelerator model (Karpetis and Varelas 2012). But, note that it is a balanced-budget constraint, not rule, since under a rule, $G - G_{-1} = T - T_{-1}$ always even though all other quantities may fluctuate; in the case of constraint, it might be that $G = T_{-1}$, and it is well known that the lag structure matters much for stabilization. And, anyway, there can be a number of ways according to which a budget is balanced (Milesi-Ferretti 2000), which might be one reason why the evidence regarding the stabilization properties of the balanced budget is mixed (Krol and Svorny 2007, vs. Levinson 2007). As the review of the literature on budgetary rules from its very beginning by Balassone and Franco (2001) suggests, the analysis becomes even more complicated and controversial when the possibility of non-balanced budget rules enter the discussion.

In principle, when income taxation is progressive, tax revenue falls more than income in a recession. And, since current consumption and investment depend on lagged and thereby already taxed income, having already the past lower taxes encouraged current consumption and investment, current balanced-budget taxation lowers the volatility of current after-tax income even more, stabilizing next period's aggregate demand even further. The linearity or nonlinearity of progression is immaterial in shaping this trend; but there may be the Fanti and Manfredi (2003) case that nonlinearity may be such that the reduction of public expenditure following the reduction in tax revenue over-offsets the increase in aggregate demand by consumption and investment. According to this paper, a linear progressive consumption tax weakens the increase of aggregate demand too, relative to income taxation, in line with the cautions about this tax raised by Boyer and Russell (1995) and mentioned in the introductory section. Although we cannot show conclusively that this weakening is so large as to be destabilizing per se, one expects that this is very likely to be the case under "Fanti-Manfredi" nonlinearity in the pursuit of tax equivalence.

In any case, stabilization under a balanced-budget rule entails an output loss relative to output in the absence of such a rule regardless type of tax. This is one aspect of the matter that has not received the proper attention by the relevant literature because of its focus on the nexus between balanced budget and output volatility. Within the context of this paper, the output loss should be ascribed to the absence of countercyclical government expenditure. Although a balanced-budget rule corroborates recovery, it does so based exclusively on the private sector depriving the economy of the countercyclical impetus of budget deficits. Consequently, in terms of volatility, the rule should be exacerbating business cycle fluctuations as reported, for example, in Kennedy and Robbins (2001). The costliness of IMF-supported stabilization programs in terms of output, owes to this precisely weakening of the public sector through the disinflation, harming subsequently the goal of international competitiveness (Hutchison and Noy 2003).

One final point that may be made in support of the stabilization properties of the consumption tax relative to the income tax, is related to internal and external habit formation in consumption. Since the consumption tax is progressive, encouraging consumers to shift consumption from the present to the future, it can correct for the inefficiency of the so-called external habit formation in consumption, viz. for intertemporal consumption externality stemming from keeping-up-with-the Joneses who care enough for their children to leave them bequests. It is an externality that leads to suboptimally large shifts of present consumption to the future, and a progressive consumption tax would make such shifts increasingly expensive, making the efficient equilibrium under external habit formation match the equilibrium under internal habit formation (Alonso-Carrera et al. 2005). Under internal habit formation in consumption, consumer's utility depends in part on how current consumption compares to past consumption, and prompts smoothing both changes and levels of consumption. An income tax can too, make the two equilibria coincide, but through a countercyclical tax rate (Alonso-Carrera et al. 2002), which is more difficult to manage than having adopted a consumption tax, because this tax operates like a fiscal rule (Kumhof and Laxton 2009), facilitating at the same time the interaction between fiscal and monetary policy (Leith and WrenLewis 2000).

Appendix

The homogeneous equation associated with [5] is:

$$x^2 - [\theta(1+\varepsilon)(1-\tau)]x + \theta\varepsilon(1-\tau) = 0$$

with solutions:

$$x_{1,2} = \frac{[\theta(1+\varepsilon)(1-\tau)] \pm \sqrt{[\theta(1+\varepsilon)(1-\tau)]^2 - 4\theta\varepsilon(1-\tau)}}{2}$$

The discriminant will be at least equal to zero iff:

$$\tau \le 1 - \frac{4\varepsilon}{\theta(1+\varepsilon)^2} < 1$$

If the discriminant is zero, i.e. if:

$$[\theta(1+\varepsilon)(1-\tau)]^2 = 4\theta\varepsilon(1-\tau) \Rightarrow$$
$$\tau = 1 - \frac{4\varepsilon}{\theta(1+\varepsilon)^2}$$

the only solution, $x = [\theta(1+\varepsilon)(1-\tau)]/2$, will be stable iff it is at most equal to 1. When the corresponding inequality is solved for τ, yields:

$$\tau \ge 1 - \frac{2}{\theta(1+\varepsilon)}$$

If the discriminant is positive, the stability condition $x \le 1$ implies for the solution with the negative square root:

$$\tau \ge 1 - \frac{\varepsilon + \sqrt{1+3\varepsilon}}{\theta(1+2\varepsilon)} \quad \text{[A1]}$$

and for the solution with the positive square root:

$$\tau \le 1 - \frac{\varepsilon + \sqrt{1+3\varepsilon}}{\theta(1+2\varepsilon)}$$

Since the denominator of [6] has to be positive:

$$\tau > 1 - \frac{1}{\theta}$$

And, since

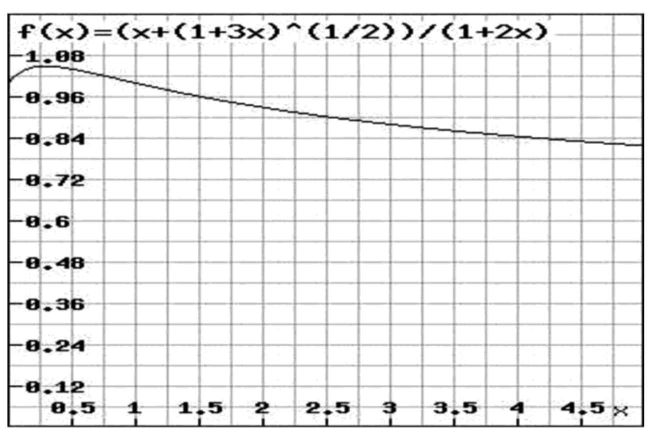
$$1 - \frac{\varepsilon + \sqrt{1+3\varepsilon}}{\theta(1+2\varepsilon)} > 1 - \frac{1}{\theta} \Rightarrow 1 + 3\varepsilon + \varepsilon^2 > 0$$

which is true, only condition [A1] and the corresponding solution are admissible. In any case, if the tax rate is to be positive and less than 1, the additional condition:

$$\frac{\varepsilon + \sqrt{1+3\varepsilon}}{(1+2\varepsilon)} < \theta$$

should hold as illustrated through Fig. A1, where $y = \theta$ (vertical axis) and $x = \varepsilon$:

Fig. A1: Combinations of $\theta \equiv f(x)$ and $\varepsilon \equiv x$



Next, the homogeneous equation associated with [11] is:

$$z^2 - \frac{\theta(1+\varepsilon)}{(1-t)} z + \frac{\theta\varepsilon}{(1-t)} = 0$$

with solutions:

$$z_{1,2} = \frac{\theta(1+\varepsilon) \pm \sqrt{[\theta(1+\varepsilon)]^2 - 4\theta\varepsilon(1-t)}}{2(1-t)}$$

The discriminant will be at least equal to zero iff:

$$t \geq 1 - \frac{\theta(1+\varepsilon)^2}{4\varepsilon}$$

If the discriminant is zero, i.e. if $[\theta(1+\varepsilon)]^2 = 4\theta\varepsilon(1-t) \Rightarrow t = 1 - \frac{\theta(1+\varepsilon)^2}{4\varepsilon}$

the only solution, $z = \theta(1+\varepsilon)/2(1-t)$, will be stable iff it is at most equal to 1 . When the corresponding inequality is solved for τ, yields:

$$t \geq 1 - \frac{\theta(1+\varepsilon)}{2}$$

If the discriminant is positive, the stability condition $z \leq 1$ implies for the solution with the positive (negative) square root that $t \leq 1 - \theta (t \geq 1 - \theta)$.

Now, consider the homogeneous equation associated with [17]:

$$u^2 - \theta(1+\varepsilon)u + \theta\varepsilon = 0$$

with solultions:

$$u_{1,2} = \frac{\theta(1+\varepsilon) \pm \sqrt{\theta^2(1+\varepsilon)^2 - 4\theta\varepsilon}}{2}$$

Fig. A2 where the vertical axis measures θ and the horizontal axis records ε, shows that the case of a discriminant equal to zero, with $\theta = 4\varepsilon/(1+3\varepsilon)$ should be ruled out as implausible; $\theta = 1$ at $\varepsilon = 1$. Stability in the case of a positive discriminant under a negative square root would imply that $1 - \theta < 0$, which is implausible as well. Therefore, only the solution with the positive square root is admissible with associated stability condition that $1 - \theta > 0$.

Fig. A2: Balanced-budget and stability: $\theta \equiv f(x)$ and $\varepsilon \equiv x$

Next, the homogeneous equation associated with [19] is:

$$v^2 - \frac{\theta(1+\varepsilon)}{1-t} v + \frac{\theta\varepsilon}{1-t} = 0$$

and the solutions are:

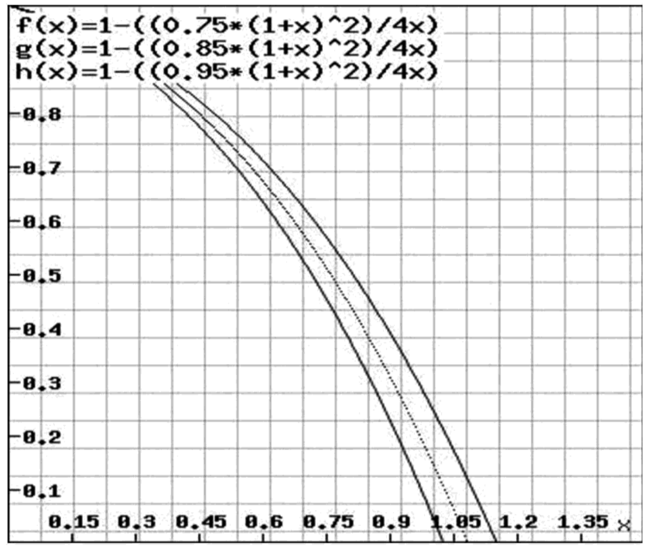
$$v_{1,2} = \frac{\theta(1+\varepsilon) \pm \sqrt{\theta^2(1+\varepsilon)^2 - 4\theta\varepsilon(1-t)}}{2(1-t)}$$

The discriminant should be positive because as Fig. $A3$ shows, only the condition:

$$t > 1 - \frac{\theta(1+\varepsilon)^2}{4\varepsilon}$$

is plausible while only the solution with the positive square root is consistent with the positive sign of the denominator of [20]. At $\varepsilon = 1$, this condition becomes: $t > 1 - \theta$

Fig. A3: Consumption tax and stability: $t \equiv y, \varepsilon \equiv x$, and $\theta = 0.75$ (higher line), $\theta = 0.85$ (middle dotted curve), and $\theta = 0.95$



Finally, $-\tau_0 + \tau\bar{Y} = -t_0 + t\bar{Y} \Rightarrow t^2 I_0(1-\theta)(1-\tau) - t[(1-\theta)(I_0 - \tau\tau_0) + \tau\theta I_0] + [\tau(1-\theta)(I_0 - \tau_0 + \theta\tau_0) + t_0 - \tau_0] = 0,$

with: $t_{1,2} = \frac{[(1-\theta)(I_0 - \tau\tau_0) + \tau\theta I_0] \pm \sqrt{\Delta}}{2I_0(1-\theta)(1-\tau)}$

where: $\Delta = [(1-\theta)(I_0 - \tau\tau_0) + \tau\theta I_0]^2 - 4I_0(1-\theta)(1-\tau)[\tau(1-\theta)(I_0 - \tau_0 + \theta\tau_0) + t_0 - \tau_0]$. If $\Delta = 0$,

the only solution would be the real one:

$$t = \frac{(1-\theta)(I_0 - \tau\tau_0) + \tau\theta I_0}{2I_0(1-\theta)(1-\tau)}$$

but the numerator would be less than the denominator so that $t < 1$, if $I_0 < \tau_0(1-\theta)$, which is simply not realistic. And, to have a positive Δ, the following condition should hold: $\tau^2(\tau_0^2 + \theta\tau_0^2 + \theta^2 I_0^2 + 4I_0^2 + 6\tau_0\theta I_0 - 4\theta I_0^2 - 4\tau_0 I_0 - 4\tau_0\theta^2 I_0) + \tau(6\theta I_0^2 + 4\tau_0\theta^2 I_0 + 4t_0 I_0 - 4I_0^2 - 2\tau_0 I_0 - 10\tau_0\theta I_0) + 1$

$(I_0^2 + \theta I_0^2 + 4\tau_0 I_0 + 4\tau_0\theta I_0 - 4t_0 I_0) > 0$. It is clear that there is no point in pursuing this matter further given that the central message of this tedious arithmetic is the nonlinearity of taxation if the two taxes are to be yielding the same revenue to the tax authority. Nevertheless, rewriting the quadratic inequality as: $\tau^2\Gamma + \tau\Lambda + \Xi > 0$, and its solutions as:

$$\tau_{1,2} > \frac{-\Lambda \pm \sqrt{\Lambda^2 - 4\Gamma\Xi}}{2\Gamma}$$

IF, $\Lambda^2 = 4\Gamma\Xi$ and hence, $\tau > -\Lambda/2\Gamma$, and IF this fraction is less than 1 , we can still conclude that: $t > \tau > -\Lambda/2\Gamma$. Consumption taxation may be nonlinear but income taxation is not, which is important because nonlinear progressive income taxation has been shown to be destabilizing.

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Footnotes

[1] Nishimura et al. (2013), and Nourry et al. (2013), address explicitly the matter of consumption taxation in connection with Schmitt-Grohe and Uribe's (1997a, 1997b, 2000) seminal work on balanced budget, taxation, and instability.

[2] More precisely, McKnight (2017) considers a version of balanced budget rule which allows for the presence of public debt as well. There is a non-balanced budget constraint, under which consumption and income taxation can have the same stability properties given time-consistent with history-dependent monetary policy, which in turn prescribes zero long-run capital taxation. But, stability, for Giannitsarou (2007), presupposes the presence of capital taxation. Moreover, a non-balanced budget constraint may be manipulated by the government so to increase its assets (increase public debt at will) until the lack of commitment is no longer binding, undermining stability regardless type of tax.

[3] Of course, investment can be a function of many factors. But, within the context of the multiplier-accelerator model, the only aspect of investment that might be investigated further is its multiplier or accelerator predominantly character. This issue has been studied by Todorova and Kutrolli (2019, 370), who finds out "that adding an accelerator coefficient reaffirms Keynesian findings, reinforcing thus the validity of the theory."

[4] A more general expression for government spending would only complicate results unnecessarily: G_0 at steady state Y, would have to be replaced by this general expression.

References

Alonso-Carrera, Jaime, Jordi Caballe and Xavier Raurich (2002), "Income Taxation with Habit Formation and Consumption Externalities", UFAE and IAE Working Paper. http://pareto.uab.es/wp/2001/49601.pdf

Alonso-Carrera, Jaime, Jordi Caballe and Xavier Raurich (2005), "Growth, Habit Formation, and Catching-up with the Joneses", *European Economic Review*, 49, 1665-1691.

Balassone, Fabrizio and Daniele Franco (2001), "EMU Fiscal Rules: A New Answer to an Old Question?",Banca d'Italia, Proceedings of the Workshop on Fiscal Rules, 2001, pp. 34-58. https://ssrn.com/abstract=2094456

Boyer, Daniel J. and Susan M. Russell (1995), "Is it Time for a Consumption Tax?",*National Tax Journal*, 48(3), 363-372.

Carroll, Robert, David Joulfaian, and James Mackie(2011), "Income Versus Consumption Tax Baselines for Tax Expenditures", *National Tax Journal*, 64 (2, Part 2), 491-510.

Fanti, Luciano and PieroManfredi (2003),"Progressive Income Taxation and Economic Cycles: A Multiplier-Accelerator Model", Discussion Papers del Dipartimento di ScienzeEconomiche – Università di Pisa, no. 11. http://www-dse.ec.unipi.it/ricerca/discussion-papers.htm

Gale, William G. and Andrew A. Samwick(2016), "Effects of Income Tax Changes on Economic Growth", Brookings Institution and Tax Policy Center Working Paper.https://www.brookings.edu/wp-content/uploads/2016/07/09_Effects_Income_Tax_Changes_Economic_Growth_Gale_Samwick_.pdf

Ghilardi and Rossi (2014), "Aggregate Stability and Balanced-Budget Rules", *Journal of Money, Credit and Banking*, 46(8), 1787-1809.

Giannitsarou, Chryssi (2007),"Balanced Budget Rules and Aggregate Instability: The Role of Consumption Taxes", *Economic Journal*, 117, 1423-1435.

Hutchison, Michael M. and IlanNoy(2003), "Macroeconomic Effects of IMF-sponsored Programs in Latin America: Output Costs, Program Recidivism and the Vicious Cycle of Failed Stabilizations", *Journal of International Money and Finance*, 22(7), 991-1014

Karpetis, Christos and ErotokritosVarelas (2012), "Fiscal and Monetary Policy Interaction in a Simple Accelerator Model", *International Advances in Economic Research*, 18(2), 199-214.

Kennedy, Suzanne and Janine Robbins (2001),"The Role of Fiscal Rules in Determining Fiscal Performance", Department of Finance Canada Working Papers 16.http:// www.fin.gc.ca/pub/pdfs/wp2001-16e.pdf

Krol, Robert and Shirley Svorny (2007), "Budget Rules and State Business Cycles", *Public Finance Review*, 35(4), 530-544.

Kumhof, Michael and Douglas Laxton (2009), "Simple, Implementable Fiscal Policy Rules", IMF Working Paper 76. https://www.imf.org/external/pubs/ft/wp/2009/wp0976.pdf

Leith, Campbell and Simon Wren-Lewis (2000), "Interactions between Monetary and Fiscal Policy Rules", *Economic Journal*, 110, C93-C108.

Levinson, Arik (2007),"Budget Rules and State Business Cycles: A Comment", *Public Finance Review*, 35(4), 545-549.

McKnight, Stephen (2017), "Are Consumption Taxes Prefe - rable to Income Taxes for Preventing Macroeconomic Instability?", *Macroeconomic Dynamics* 21, 1023-1058.

Meng, Qinglai and JianpoXue(2015), "Balanced-Budget Con - sumption Taxes and Aggregate Stability in a Small Open Economy", *Economic Letters*, 137, 214-217.

Milesi-Ferretti, Gian Maria (2000), "Good, Bad or Ugly?On The Effects of Fiscal Rules with Creative Accounting", IMF Working Paper 172.https://www.imf.org/external/pubs/ft/wp/2000/wp00172.pdf

Nishimura, Kazuo, CarineNourry, Thomas Seegmuller and Alain Venditti(2013), "Destabilizing Balanced-Budget Consumption Taxes in Multi-sector Economies", *Interna - tional Journal of Economic Theory*, 9(1), 113-130.

Nourry, Carine, Thomas Seegmuller and Alain Vendit - ti(2013),"Aggregate Instability under Balanced-Budget Consumption Taxes: A Re-examination", *Journal of Eco - nomic Theory*, 148(5), 1977-2006.

Ortigueira, Salvador (2012), "Markov-perfect Optimal Fiscal Policy: The Case of Unbalanced Budgets", Univer - sidad Carlos III de MadridUC3M Working papers.https:// e-archivo.uc3m.es/bitstream/handle/10016/15788/we1230.pdf?sequence=1

Schmitt-Grohe, Stephanie and Martin Uribe (1997a), "Balanced-Budget Rules and Macroeconomic (in)Stabi - lity", *Journal of Economic Theory*, 119(2), 357-363.

Schmitt-Grohe, Stephanie and Martin Uribe (1997b), "Balan

- ced-Budget Rules, Distortionary Taxes, and Aggregate Instability", *Journal of Political Economy*, 105(5), 976-1000.

Schmitt-Grohe, Stephanie and Martin Uribe (2000), "Price Level Determinacy and Monetary Policy under a Balan - ced-Budget Requirement", *Journal of Monetary Econo - mics*, 45(1), 211-246.

Todorova, Tamara P. and Marin Kutrolli (2019), "An Expan - ded Mltiplier-Accelerator Model", *Journal of Economics and Political Economy*, 6(4), 368-382.

Turnovsky, Stephen J. (1977), *Macroeconomic Analysis and Stabilisation Policy*, Cambridge: Cambridge University Press.

Westerhoff, Frank H. (2006), "Samuelson's Multiplier-Ac - celerator Model Revisited, *Applied Economic Letters*, 13(2), 89-92.